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ON THE DIGITS OF NUMBERS IN THE SYSTEM LOGIC B_3

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Abstract. This study is about digits of numbers in system logic B_3. Any real number is written as digits in the binary system, in the ternary system. The numbers in base two and base three are also written in the B_3 system ternary logic. These two writing methods are transferred into the third method. The real numbers 0,1 and $0, 1, 2$ are written as digits. The same real numbers are written as digits of elements of the set $-1, 0, 1$ in base B_3. The periods here are investigated. The relationship between these digits is analysed.

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1. Introduction

The intensive studies on this subject are carried out in the 16^{th}, 17^{th}, 18^{th} and current centuries. The binary number system is used in many societies. For example, Egypt, China and India are the leading societies that use this system. Harriot, Lobkowitz and Leibniz studied the subject at the same time. Leibniz's contribution to this subject is started with the "Yi Jing" written in Chinese. The binary number system is used until today [5, 6]. The binary system is gained momentum with Cayleyin's discovery of idempotent elements. This topic is discussed by Boolean and De Morgan [1, 3]. Aliyev is studied the digits of powers of 2 of a real number in triple base in [11]. This system is created the many technological field for itself [4, 7, 9].

In general, an integer k is given in base a as follows.

$$k = \sum_{i=0}^{n} c_i a^i, \;\; where\ c_i \in \{0, 1, \ldots, a-1\}. \tag{1}$$

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An integer k can be written as powers of 2 or 3, that is

If $a = 2$, then

$$k = \sum_{i=0}^{n} c_i 2^i, \ \textit{where } c_i \in \{0, 1\}. \tag{2}$$

If $a = 3$, then

$$k = \sum_{i=0}^{n} c_i 3^i, \ \textit{where } c_i \in \{0, 1, 2\}. \tag{3}$$

If the identity element of the second operation in a binary structure is 1, then the inverse element of this element with respect to the first operation is -1.

$$1 + (-1) = 0.$$

Any integer in the system logic B_3 is written in both base 3 and base 2. Any integer is not written in base 3 using $\{0, 1\}$ in B_2. For example,

$$5 = \sum_{i=0}^{2} c_i 2^i, \ \textit{where } c_i \in \{0, 1\}. \tag{4}$$

,

$$5 = (12)_3. \tag{5}$$

$$5 = \sum_{i=0}^{3} c_i 3^i, \ \textit{where } c_i \in \{-1, 0, 1\}. \tag{6}$$

$$5 = \sum_{i=0}^{3} c_i 2^i, \ \textit{where } c_i \in \{-1, 0, 1\}. \tag{7}$$

2. Main results

Numbers written in a base are the sum of the powers of that base. The definition of centroid is given to better see the relationship between the numbers in the base. Let us start with the following definition of centroid.

Definition 2.1. If an integer k can be written $k = ba^s$ in terms of a base a, a constant b and a positive integer s, then k is called a centroid on base $a > 1$, in [2, 8, 10, 12].

The writing of an integer $k > 0$ in logic B_i and in base a is denoted by $(B_i)_a$ and in logic base B_i by (base $a)_{B_i}$, where $i = 2, 3$. This integer k is the base of the centroid. The integer s is called the period of centroid and it is denoted by $p((B_i)_a(k))$. That is

$$p((B_i)_a(k)) = s \tag{8}$$

The set of all elements formed by a centroid in base a in logic B_i is denoted by $C_{(B_i)_a}(a^s)$. The number of digits a integer k in base a in logic B_i is denoted by $d_{(B_i)_a}(k)$. By the definitions of centroid and using the given $d_{(B_i)_a}(k)$, we get

$$C_{(B_i)_a}(a^s) = \{k \mid d_{(B_i)_a}(a^s) = d_{(B_i)_a}(k)\}. \tag{9}$$

TABLE 1. Their equivalents in $\mathcal{B}_2$ and $\mathcal{B}_3$ in base 2 for some numbers.

Numbers	Base 2		Centroid	
	B_2	B_3	2^s	$C(_{B_2})_2(2^s)$
1	(1)	$(1-1)$	2^0	$\{1\}$
2	(10)	$(1-10)$	2^1	$\{2,3\}$
3	(11)	$(1-1-1)$		
4	(100)	$(1-100)$		
5	(101)	$(10-1-1)$	2^2	$\{4,5,6,7\}$
6	(110)	$(10-10)$		
7	(111)	$(100-1)$		
8	(1000)	$(1-1000)$		
9	(1001)	$(1-1001)$		
10	(1010)	$(1-1010)$		
11	(1011)	$(1-1011)$		
12	(1100)	$(1-1100)$	2^3	$\{8,9,10,11,12,13,14,15\}$
13	(1101)	$(1-1101)$		
14	(1110)	$(1-1110)$		
15	(1111)	$(1-1111)$		
16	(10000)	$(1-10000)$		
17	(10001)	$(1-10001)$		
18	(10010)	$(1-10010)$		
19	(10011)	$(1-10011)$		
20	(10100)	$(1-10100)$		
21	(10101)	$(1-10101)$	2^4	$\{16,17,18,\cdots,29,30,31\}$
22	(10110)	$(1-10110)$		
23	(10111)	$(1-1100-1)$		
24	(11000)	$(1-11000)$		
25	(11001)	$(1-11001)$		
26	(11010)	$(1-11010)$		
27	(11011)	$(1-11010)$		
28	(11100)	$(1-11100)$		
29	(11101)	$(1-11101)$		
30	(11110)	$(1-11110)$		
31	(11111)	$(1-11111)$		
32	(100000)	$(1-100000)$		
$\vdots$	$\vdots$	$\vdots$	2^5	$\{32,\cdots,63\}$
63	(111111)	$(1-100000)$		

Some examples of centroid ones are given below.

Example 2.2. *The centroids of* $2, 4$ *and* 8 *in logic* B_2 *in base* 2.

$$C(_{B_2})_2(2) = \{2, 3\}, \tag{10}$$

$$C(_{B_2})_2(2^2) = \{4, 5, 6, 7\}, \tag{11}$$

$$C(_{B_2})_2(8) = \{8, 9, 10, 11, 12, 13, 14, 15\}. \tag{12}$$

Proposition 2.3. *If an integer* $k > 0$*, then*

$$k = (B_2)_2 = (base\ 2)_{B_2}$$

Proof. If an integer $k > 0$ and The number is written in base two in terms of 0 and 1, then

$$k = (B_2)_2 = (\text{base } 2)_{B_2}$$

. □

Proposition 2.4. *If an integer* $k > 0$*, then*

$$k = (B_2)_2 k = (B_3)_2 (B_2)_2 \neq (B_3)_2$$

Proof. If an integer $k > 0$ and the number is written in base two in terms of 0 and 1 and the same number is written in base 2 according to B_3, then this software is composed of $0s$, $1s$ and $2s$ in base 2, then

$$k = (B_2)_2$$

$$k = (B_3)_2$$

and

$$(B_2)_2 \neq (B_3)_2.$$

□

Remark 2.1. If two expressions are equal to one expression, then these two expressions are equal to each other in logic B_2.

Proposition 2.5. *If an integer* $k > 0$*, then*

$$(B_3)_3 \neq (base\ 3)_{B_3}$$

.

Example 2.6. *For* $k = 2$

$$2 = (02)_3, \tag{13}$$

$$2 = (1 - 1)_{B_3}. \tag{14}$$

But,

$$(02)_3 \neq (1 - 1)_{B_3}$$

Remark 2.2. If two expressions are equal to the same expression, then these two expressions are not equal to each other in logic B_3.

3. The Digits of Numbers in the System Logic B_3

In this section, the digit numbers of a number written in logic B_3 are analysed. the period is composed to the number of $0s$ and $1s$ in logic B_2. The period is composed to the number of $-1s$, $0s$ and $1s$ in logic B_3. In the following, the software of some numbers in different bases in logics B_2 and B_3 are analysed.

$$4 = (10)_4$$

$$4 = (B_2) = (10)_4 = (B_3)_4, p((B_3)_4(4)) = p((B_2)_4(4)) = 2$$

$$5 = (B_2) = (11)_4 = (B_3)_4, p((B_3)_4(5)) = p((B_2)_4(5)) = 2$$

The number 12 is not writable in base 4 in logic B_2 and $p((B_2)_4(12)) = 0$. But,

$$12 = (1-10)_4 = (B_3)_4, p((B_3)_4(12)) = 3$$

.

Lemma 3.1. *If $a = 2$, then $C(_{B_2})_2(2^s) = C(_{B_3})_2(2^s)$, where $s \in \mathbb{Z}^+$.*

Proof. For any $k \in C(_{B_2})_2(2^s)$, $d(_{B_2})_2(2^s) = d(_{B_2})_2(k)$ and by equation 9 we get $d(_{B_3})_2(2^s) = d(_{B_3})_2(k)$ and $k \in C(_{B_3})_2(2^s)$,

$$C(_{B_2})_2(2^s) \subseteq C(_{B_3})_2(2^s) \tag{15}$$

Similar method is used for

$$C(_{B_3})_2(2^s) \subseteq C(_{B_2})_2(2^s) \tag{16}$$

By equation 15 and equation 16, we have

$$C(_{B_2})_2(2^s) = C(_{B_3})_2(2^s).$$

□

Lemma 3.2. *For any $s \in \mathbb{Z}^+$ if $C(_{B_2})(2^s)$ is a centroid, then*

$$\mid C(_{B_2})_2(2^s) \mid = 2^s.$$

Proof. The number of numbers between integers 2^{s+1} and 2^s is

$$\frac{2^{s+1} - 2^s}{1} = \frac{2^s(2-1)}{1} = 2^s.$$

$$\mid C(_{B_2})_2(2^s) \mid = 2^s.$$

□

The periods of numbers in logic B_3 are investigated in the continuation of this study.

Theorem 3.3. *For any $s \in \mathbb{Z}^+$ if $C(_{B_2})(2^s)$ is a centroid, then the followings hold.*

(i) $p((B_2)_2(2^s)) = p((B_3)_2(2^s)) = s$.
(ii) $d(_{B_2})_2(2^s) = s + 1$.
(iii) $d(_{B_3})_2(2^s) = s + 2$.

Proof. For any $s \in \mathbb{Z}^+$ if $C(_{B_2})(2^s)$ is a centroid, then

(i) The proof(i) is clear by definition 2.1.

(ii) we have $2^s = \sum_{i=0}^{s} c_i 2^i$, where $c_s = 1$ and all other c_i are zero.

$$(\underbrace{10...0}_{(s+1)-times})$$

We get

$$d(_{B_2})_2(2^s) = s + 1.$$

(iii) In B_3, we have $2^s = \sum_{i=0}^{s+1} c_i 2^i$, , where $c_{s+1} = 1$, $c_s = -1$ and all other c_i are zero.

$$(\underbrace{1-10...0}_{(s+2)-times})$$

We get

$$d(_{B_3})_2(2^s) = s + 2.$$

$\square$

4. Conclusions and Discussions

All numbers are written in terms of 0 and 1 in digital systems. The inverse of 1 is the number -1. This -1 number is included in the logic system. Any positive integer is written with -1, 0 and 1 in base 2. And the numbers 0 and 1 are in B_3 in the base 2. If k is any positive integer, this number is $k = (10)_k$ according to logic B_2 in base k. Many positive integers cannot be written in different bases according to logic B_2. For example, 7 cannot $(B2)_4$ be written. In this study, all numbers written in base 2 according to logic B_2 are also written in the same base according to logic B_2.

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